Math in depth

Mathematics from high school to university

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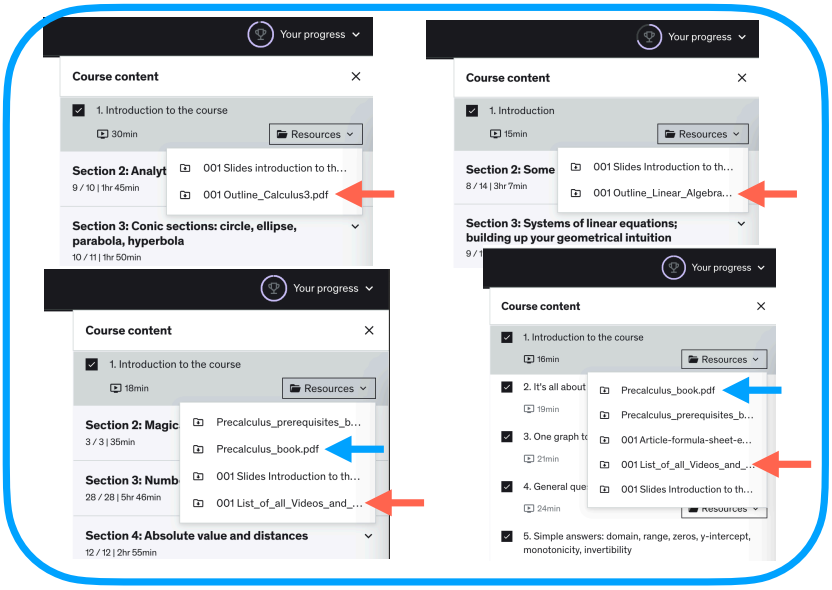
How to get more practice, and which books to read along with our courses

We often get questions about which **textbooks** to read along with our courses, and about possibilities of **getting more practice** while studying the subjects we cover. I (Hania) will answer these questions here. This document will be placed in the **Bonus Section** in all our courses, and it will be updated each time we release a new course.

1. How to get more practice

- We don't have any quizzes in our courses, because we don't believe that taking quizzes is the right way to test your mathematical knowledge.
- Our courses contain plenty of solved problems. A majority of them are solved in videos, and some of them are solved in attached articles. You can see the texts of all the problems listed in an outline document attached as a resource to Video 1 (in each course; see some examples in the screenshots beneath).



Figur 1: Where to find the resources described in this document: 1. The **outline** (list of all videos and problems; called **outline** in our older courses, and called **List of all Videos and Problems** in our later courses) is attached as a resource to Video 1 in all our courses; 2. **Precalculus book**: same place, in **Precalculus** series. The screenshots show: *Calculus* 3*, part* 1 *of* 2, *Linear Algebra and Geometry* 1, *Precalculus* 1*: Basic notions*, and *Precalculus* 4*: Exponentials and logarithms.*

> You can choose whether to watch me solve the problems, or try to solve them on your own. You can also watch the entire course first, and afterwards go back to the list and solve all the problems in order to verify that you really master all the topics. You know where to find the outlines, and we hope that you will discover how helpful they are.

- You can find more exercises for practice in the textbooks mentioned in the next section, or in other textbooks covering the same or related topics.

For ALL maths courses, the most wonderful source of problems with solutions are SCHAUM'S OUTLINES. You can get them for advanced calculus and other topics; there are different versions and editions: all of them are absolutely great.

2. Books to read along with our courses

Our courses are self-contained—both theory and practice are covered—so you don't *need* any additional books. Of course, it can be nice to have some book to read along, and this is why I will make some suggestions here. Our courses are based on everything I have learned during my studies in Poland (mathematical class in high school, and Master Program in Theoretical Mathematics at Copernicus University in Toruń) and in Sweden (PhD in Applied Discrete Mathematics at Uppsala University), and completed by my later experience as a university teacher (both in Belgium and in Sweden). During my time as a lecturer I worked a lot with illustrations, geometrical interpretations, intuitive explanations, and various types of software (GeoGebra, MANIM, LaTeX, TikZ,...) in teaching. I believe that this combination of the old Polish way of teaching with modern ideas and tools makes our courses both formally correct and intuitively understandable.

None of our courses are based on a book, but in one of our series (**Precalculus**) I chose a free on-line book to recommend to my students **before** I started to create the series. This is why you get reading recommendations in the **outline** documents, and suggestions of problems to solve in some **videos**. See some examples of such recommendations given in the course *Precalculus* 3:

S9 Basic properties of six trigonometric (circular) functions; graphing

You will learn: the definition of the other circular functions (tangent, and the three reciprocals) defined with help of sine and cosine; basic properties following immediately from the definitions and symmetries of the unit circle: the domain and range for all these functions, Reference Angles Identities, monotonicity in intervals, being even or odd, periodicity (a new concept, not introduced in Precalculus 1), the graphs; basic relationships between these functions: the Pythagorean Identity, cofunction identities. You will also learn the etymology of the names sine, tangent, and secant.

Read along with this section: **Precalculus book**: Section 10.2 (pp. 717–743); you have probably already seen some parts of this section while studying our Section 8. Observe that my approach is different than the approach of the Authors: I started with geometry, they started with the functional approach. This means that the part 10.2.1 *Beyond the unit circle* will not really be necessary for us, as we treated all the ratio-related issues much earlier in the course (Sections 2 and 4). Section 10.3 (pp. 744–769) and some parts of Section 10.5 (pp. 790–818): only about the graphs of the six circular functions, **not yet** about their transformations. You can also omit the exercises about solving trigonometric equations, as we will deal with them in Section 13.

Prec3, Video 105

Page 809, Answers on page 818

In Exercises 25 - 34, use Example 10.5.3 as a guide to show that the function is a sinusoid by rewriting it in the forms $C(x) = A\cos(\omega x + \phi) + B$ and $S(x) = A\sin(\omega x + \phi) + B$ for $\omega > 0$ and $0 \le \phi < 2\pi$.

25. $f(x) = \sqrt{2}\sin(x) + \sqrt{2}\cos(x) + 1$
26. $f(x) = 3\sqrt{3}\sin(3x) - 3\cos(3x)$
27. $f(x) = -\sin(x) + \cos(x) - 2$
28. $f(x) = -\frac{1}{2}\sin(2x) - \frac{\sqrt{3}}{2}\cos(2x)$
29. $f(x) = 2\sqrt{3}\cos(x) - 2\sin(x)$
30. $f(x) = \frac{3}{2}\cos(2x) - \frac{3\sqrt{3}}{2}\sin(2x) + 6$
31. $f(x) = -\frac{1}{2}\cos(5x) - \frac{\sqrt{3}}{2}\sin(5x)$
32. $f(x) = -6\sqrt{3}\cos(3x) - 6\sin(3x) - 3$

810 FOUNDATIONS OF TRIGONOMETRY

33. $f(x) = \frac{5\sqrt{2}}{2}\sin(x) - \frac{5\sqrt{2}}{2}\cos(x)$
34. $f(x) = 3\sin\left(\frac{x}{6}\right) - 3\sqrt{3}\cos\left(\frac{x}{6}\right)$

Prec3, Video 131

Pages 784

In Exercises 59 - 73, verify the identity. Assume all quantities are defined.

59. $(\cos(\theta) + \sin(\theta))^2 = 1 + \sin(2\theta)$
60. $(\cos(\theta) - \sin(\theta))^2 = 1 - \sin(2\theta)$
61. $\tan(2\theta) = \frac{1}{1-\tan(\theta)} - \frac{1}{1+\tan(\theta)}$
62. $\csc(2\theta) = \frac{\cot(\theta) + \tan(\theta)}{2}$
63. $8\sin^4(\theta) = \cos(4\theta) - 4\cos(2\theta) + 3$
64. $8\cos^4(\theta) = \cos(4\theta) + 4\cos(2\theta) + 3$
65. $\sin(3\theta) = 3\sin(\theta) - 4\sin^3(\theta)$
66. $\sin(4\theta) = 4\sin(\theta)\cos^3(\theta) - 4\sin^3(\theta)\cos(\theta)$
67. $32\sin^2(\theta)\cos^4(\theta) = 2 + \cos(2\theta) - 2\cos(4\theta) - \cos(6\theta)$
68. $32\sin^4(\theta)\cos^2(\theta) = 2 - \cos(2\theta) - 2\cos(4\theta) + \cos(6\theta)$
69. $\cos(4\theta) = 8\cos^4(\theta) - 8\cos^2(\theta) + 1$
70. $\cos(8\theta) = 128\cos^8(\theta) - 256\cos^6(\theta) + 160\cos^4(\theta) - 32\cos^2(\theta) + 1$ (HINT: Use the result to 69.)
71. $\sec(2\theta) = \frac{\cos(\theta)}{\cos(\theta) + \sin(\theta)} + \frac{\sin(\theta)}{\cos(\theta) - \sin(\theta)}$
72. $\frac{1}{\cos(\theta) - \sin(\theta)} + \frac{1}{\cos(\theta) + \sin(\theta)} = \frac{2\cos(\theta)}{\cos(2\theta)}$
73. $\frac{1}{\cos(\theta) - \sin(\theta)} - \frac{1}{\cos(\theta) + \sin(\theta)} = \frac{2\sin(\theta)}{\cos(2\theta)}$

Here are my suggestions for textbooks:

1. The **Precalculus** series:

   Books to read along with the course, with more practice problems (often suggested in the outline, and in some videos, as I have found these books before I started to record the series):

   I *Precalculus Prerequisites, a.k.a. Chapter* 0: Carl Stitz, Ph.D., Lakeland Community College; Jeff Zeager, Ph.D., Lorain County Community College; version from August 16, 2013.

   II *Precalculus*: Carl Stitz, Ph.D., Lakeland Community College; Jeff Zeager, Ph.D., Lorain County Community College; version from July 4, 2013.

   These books are added as resources to Video 1 in each part of the Precalculus series, with kind permission of Professor Carl Stitz.

2. The **Linear-Algebra-and-Geometry** series:

   I recommend the following books:

   1. `https://math.emory.edu/~lchen41/teaching/2020_Fall/Nicholson-OpenLAWA-2019A.pdf`
   2. Gilbert Strang: *Introduction to linear algebra.*
   3. David C. Lay: *Linear Algebra and Its Applications.*

   There are no reading recommendations in this series, as I chose these books *after* having recorded the series, so just look in the table of contents, and choose the topics you want to read more about, or practice with. One exception: some reading recommendations from the third book are given in *Linear Algebra and Geometry* 3, in Video 49, as I used some examples from this book in Section 5.

3. The **Calculus** series:

   There are plenty of good Calculus books available in English, and many of them are great. My favourite is:

   Robert A. Adams, Christopher Essex: *Calculus, a complete course.* 8th or 9th edition.

   In our *Calculus* 3 (which was our very first one on Udemy) I wrote section numbers from this book in both the **outline** and the title pages of all the presentations, so, in a way, you got some reading recommendations from this book in *Calculus* 3.
   We haven't created **Calculus 1** and **Calculus 2** subseries yet, but I was looking for a free textbook in **Calculus**, and I have found this one:

   Gilbert Strang & Edwin *Jed* Herman: **Calculus**, OpenStax:

   `https://math.libretexts.org/Bookshelves/Calculus/Calculus_(OpenStax)`

   You can get the same book as a pdf (in one piece) if you use the red button PDF→Full Book on the page under the link given above. The book is also added as a resource to Video 1, so that we can be sure that the reading recommendations agree with the book you get, regardless possible changes performed in the book available online. We got a permission to do so from LibreTexts Office on July 20th, 2023. We don't sell the book, and the price of our courses is not affected by the presence of this book in resources; the book is available for everybody under the link given above. You can view the licence here:

   `https://math.libretexts.org/Bookshelves/Calculus/Calculus_(OpenStax)/zz%3A_Back_Matter/30%3A_Detailed_Licensing`

   `https://creativecommons.org/licenses/by-nc-sa/4.0/`

You can read this book along with our **Calculus** series if you wish to, but, again, this is not really necessary as our courses are self-contained. This is how the chapters of this book correspond (more or less) to the content of our courses:

Calculus 1, part 1 of 2: Limits and continuity

https://math.libretexts.org/Bookshelves/Calculus/Calculus_(OpenStax)/09%3A_Sequences_and_Series

https://math.libretexts.org/Bookshelves/Calculus/Calculus_(OpenStax)/02%3A_Limits

Calculus 1, part 2 of 2: Derivatives with applications

https://math.libretexts.org/Bookshelves/Calculus/Calculus_(OpenStax)/03%3A_Derivatives

https://math.libretexts.org/Bookshelves/Calculus/Calculus_(OpenStax)/04%3A_Applications_of_Derivatives

Calculus 2, part 1 of 2: Integrals with applications

https://math.libretexts.org/Bookshelves/Calculus/Calculus_(OpenStax)/05%3A_Integration

https://math.libretexts.org/Bookshelves/Calculus/Calculus_(OpenStax)/06%3A_Applications_of_Integration

https://math.libretexts.org/Bookshelves/Calculus/Calculus_(OpenStax)/07%3A_Techniques_of_Integration

Calculus 2, part 2 of 2: Sequences and series

https://math.libretexts.org/Bookshelves/Calculus/Calculus_(OpenStax)/09%3A_Sequences_and_Series

https://math.libretexts.org/Bookshelves/Calculus/Calculus_(OpenStax)/10%3A_Power_Series

Calculus 3, part 1 of 2

https://math.libretexts.org/Bookshelves/Calculus/Calculus_(OpenStax)/11%3A_Parametric_Equations_and_Polar_Coordinates

https://math.libretexts.org/Bookshelves/Calculus/Calculus_(OpenStax)/12%3A_Vectors_in_Space

https://math.libretexts.org/Bookshelves/Calculus/Calculus_(OpenStax)/13%3A_Vector-Valued_Functions

https://math.libretexts.org/Bookshelves/Calculus/Calculus_(OpenStax)/14%3A_Differentiation_of_Functions_of_Several_Variables

Calculus 3, part 2 of 2

https://math.libretexts.org/Bookshelves/Calculus/Calculus_(OpenStax)/15%3A_Multiple_Integration

https://math.libretexts.org/Bookshelves/Calculus/Calculus_(OpenStax)/16%3A_Vector_Calculus

4. **Real Analysis and advanced Calculus**:
   the *Lecture Notes from UC Davis* by Professor John K. Hunter:

   https://www.math.ucdavis.edu/~hunter/intro_analysis_pdf/intro_analysis.html

Remember that you can always ask questions on Q&A!